Astro 501 Final

This test is open notes. Answer all questions and PLEASE BE NEAT. If possible, write out the formulae before plugging in the numbers. If the questions require assumptions, state and justify them clearly! Here are some constants that may (or may not be useful):

1. A mass exchange binary, consisting of an M-dwarf star accreting onto a white dwarf, has coordinates of $\alpha(2000)$ =0:30:00, $\delta(2000)$ =+41:00:00.

a) (5 pts) Roughly speaking, at what time tonight will the object be on the meridian?

On the autumnal equinox, the sidereal time equals the solar time; after that, the sky progresses about two hours per month, or roughly 1 hour per week. Since we are one week from the winter solstice, at midnight the right ascension will be about 5:30. An object at 0:30 will therefore transit five hours earlier, or about 7:00 p.m.

b) (10 pts) Tonight, astronomical twilight begins about 6 p.m. local standard time. How long will the object be observable (i.e., above two airmasses) this evening? Assume that the latitude of State College is $\phi = 40.8°$.

Airmass is given by

$$M = \sec z\{1 - 0.0012(\sec^2 z - 1)\} \approx \sec z \implies \sec z = 2 \implies z \sim 60°$$

To get the hour angle that this corresponds to, one can use

$$\sin a = \sin\delta \sin\phi + \cos\delta \cos H \cos\phi$$

where the altitude $a = 90 - z$. So

$$\cos H = \frac{\sin a - \sin\delta \sin\phi}{\cos\delta \cos\phi} = 0.138$$

This implies $H = 82°$ or 5.5 hours. Since we cannot start observing until 6 p.m., the object rises for an hour, crosses the the meridian, and sets for 5.5 hours until it reaches 2 airmasses. It is therefore observable for 6.5 hours.

c) (10 pts) Suppose the binary system were to undergo an outburst and reach an apparent magnitude of $m = 6$. If the object is at a distance of ~ 5 kpc, can we conclude that mass is being lost from the system?

If the object is at a distance of 5 kpc, then its distance modulus is

$$\mu = 5 \log d - 5 = 13.5$$

its absolute magnitude is

$$M = m - \mu = -7.5$$

and its absolute luminosity is

$$\mathcal{L} = 10^{(C-M)/2.5} = 80000\mathcal{L}_\odot = 3 \times 10^{38} \text{ ergs s}^{-1}$$

where $C = 4.75$ is the absolute magnitude of the Sun. We can compare this to the Eddington luminosity of a $1.4M_\odot$ object,

$$\mathcal{L}_{Edd} = \frac{4\pi c G\mathcal{M}}{\kappa} = 1.4 \times 10^{38}\mathcal{M} = 2 \times 10^{38} \text{ ergs s}^{-1}$$

Since the luminosity of the outburst is greater than Eddington luminosity, mass must be being lost from the system.

2. (10 pts) The Super-Kamiokande experiment is designed to detect solar neutrinos produced by the PP III branch of the proton-proton chain. The experiment finds that the PP III neutrino flux at the earth is 2×10^6 neutrinos cm^{-2} s^{-1}. If the PP III branch produces $\sim 0.01\%$ of the neutrinos emitted by the Sun, how much of discrepancy is there between observed neutrino flux and the predicted flux.

The process of fusing hydrogen to helium produces 26.73 MeV per helium nucleus, and, in the formation of helium nucleus, 2 neutrinos are produced. Thus, the number of neutrinos that the Sun is producing each second is

$$N_\nu = \mathcal{L}_\odot \left(\frac{1 \text{ MeV}}{1.6 \times 10^{-6} \text{ ergs}}\right)\left(\frac{\text{He}}{26.73 \text{ MeV}}\right)\left(\frac{2}{\text{He}}\right) = 1.8 \times 10^{38} \text{ neutrinos/sec}$$

where $\mathcal{L}_\odot$ is the luminosity of the Sun in ergs s^{-1}. The neutrino flux at earth ($d = 1.5 \times 10^{13}$ cm) is then

$$f_\nu = \frac{N_\nu}{4\pi d^2} = 6.6 \times 10^{10} \text{ neutrinos cm}^{-2} \text{ s}^{-1}$$

Just 0.0001 of these neutrinos are PP III neutrinos, so

$$f_\nu(\text{PPIII}) = 0.0001 f_\nu = 6.3 \times 10^6 \text{ neutrinos cm}^{-2} \text{ s}^{-1}$$

or ~ 3.1 times the observed flux.

3. (10 pts) A star with a spectral type G2 V is observed to be a single-line spectroscopic binary. The radial velocity curve is sinusoidal (implying that the orbit is circular) with a period of 3 hours. Show that the unseen companion is much more likely to be a white dwarf than a main sequence star.

Newton's modification of Kepler's 3rd law is

$$a^3 = (M_1 + M_2)P^2$$

where M is in solar masses, P is in years, and a is in Astronomical Units. Since the primary is a G2 V main sequence star, its mass is $1M_\odot$, so the semi-major axis of the orbit must be less than 0.005 A.U., or $1.05R_\odot$. Since the G2 V star should have a radius of $1R_\odot$, the second star must have a radius less than $0.05R_\odot$. While some M main sequence stars and brown dwarfs are this small, the more likely answer is that the unseen companion is a white dwarf.

4. (10 pts) In a low-density ionized nebula, about three quarters of all electrons recombining into hydrogen eventually enter the 2P state, and then decay to 1S via Lyα; the other 25% fall into 2S and then decay to 1S via 2-photon emission. Hence roughly three quarters of all recombinations create Lyα. But what about higher-density regions? Here are the quantum mechanical collision strengths for the two most common transisions out of the 2S state. (For simplicity, I've combined the 3S, 3P, and 3D terms.)

State	Ω
$2S \rightarrow 1S$	0.26
$2S \rightarrow 2P$	6153
$2S \rightarrow 3(S+P+D)$	25.6

If the Einstein coefficient for 2-photon emission is A = 8.23 sec^{-1}, at what density does 2-photon emission begin to become suppressed and the fraction of Lyα photons per recombination increase? (Assume a 10,000 K plasma.)

If N_i is the number of H I ions in $2S$ state, then collisions become more important than radiative decays when

$$\sum_{j \neq i} N_i N_e \, q_{ij} > \sum_{j<i} N_i A_{ij}$$

or

$$N_e > \frac{\sum_{j<i} A_{ij}}{\sum_{j \neq i} q_{ij}}$$

where the collision strength q_{ij} is given by

$$q_{ij} = 8.629 \times 10^{-6} \frac{\Omega_{ij}}{\omega_i T_e^{1/2}} e^{-\Delta E/kT}$$

where the Boltzmann factor is only applicable to upward collisions. (A quick inspection of the problem shows that since the Omega value for the 2S-2P transision is much greater than that for 2S-1S, the latter can be ignored. Moreover, upward collisions are suppressed by the Boltzmann factor, so they are also small. So really, all one has to deal with is the one 2S-2P rate.) Plugging in the numbers gives a critical density of 1.5×10^4 cm^{-3}. At higher densities, 2-photon emission is suppressed, and Lyα is enhanced.

5. (10 pts) In the immediate vicinity of M31's black hole (i.e., within ~ 0.01 pc), the stellar density is $\sim 10,000,000$ stars per cubic parsec, and the velocity dispersion is 400 km s^{-1}. Should we consider putting in a telescope proposal to monitor the region for stellar collisions?

The typical timescale for a star's orbit to be substantially perturbed by interactions with other stars is given by the relaxation time

$$T_p = \frac{\nu^3}{8\pi G^2 M^2 N} \ln\left\{\pi \frac{b_{\mathrm{max}}\,\nu^2}{2GM}\right\} = 2.1 \times 10^9 \frac{\nu^3}{M^2 N} \ln\left\{365\left(\frac{b_{\mathrm{max}}\,\nu^2}{M}\right)\right\} \text{ yr}$$

where the velocity is given in kilometers per second, the typical stellar mass in $M_\odot$, the density in stars in pc^{-3}, the stellar density, N is in stars per cubic parsec, and, very roughly,

$$b_{\mathrm{max}} = \left(\frac{3}{4\pi N}\right)^{1/3}$$

Plugging in the numbers gives a relaxation time of $\sim 10^{10}$ years. In other words, it takes about a Hubble time for a star's orbit to be gravitationally perturbed by another star. The odds of a stellar collision is extremely low.

6. (10 pts) As we saw in class, the effect of Baryonic Accoustic Oscillations in the early universe creates a small blip in the 2-point correlation function: when one analyzes the distribution of hot/cold spots in the microwave background, one finds that the characteristic BAO scale is 1.6 degrees. That scale has been frozen into the universe ever since. For a flat, $\Lambda = 0$ (Einstein-de Sitter) cosmology with $H_0 = 70$ km s^{-1} Mpc^{-1}, what is the scale of those fluctuations (in Mpc) today? (Assume the microwave background was released at $z \sim 1500$.)

The angular size of an object at redshift z is related to its physical size, s, by

$$\theta = \frac{s(1+z)}{d_p} \Longrightarrow s = \frac{\theta d_p}{(1+z)} = \frac{2\theta c}{H_0}\left\{1 - (1+z)^{-1/2}\right\}$$

Plugging in the numbers gives a characteristic distance of 233 Mpc.

7. Bill Harris' globular cluster catalog lists the following information for the cluster M13:

Central Surface Brightness (mag arcsec^2)	$\mu_V = 16.6$
Line-of-Sight Velocity Dispersion (km/s)	$\sigma = 7.1$
Core Radius (arcmin)	$r_0 = 0.62$
Distance (pc)	$d = 7350$
Total Brightness (mag)	$V = 5.78$
V-band Extinction (mag)	$A_V = 0.06$
Concentration ($\log r_t/r_0$)	$c = 1.53$
Galactic-centric distance (pc)	$d = 8400$

a) (10 pts) What is the mass to light ratio of the cluster in solar units? Is there evidence for dark matter in the cluster?

At a distance of 7350 pc, 1 arcsec corresponds to $d \sin\theta \approx d\theta = 0.036$ pc, so a core radius of 0.62 arcmin corresponds to 1.3 pc. For an isothermal sphere, the central density is therefore

$$\rho_0 = \left(\frac{9\sigma^2}{4\pi G r_0^2}\right) = \frac{\Sigma_0}{2.018\, r_0}$$

After plugging in the numbers, the projected surface mass is then

$$\Sigma_0 = 2.018 \frac{9\sigma^2}{4\pi G r_0} = 12850 M_\odot \text{ pc}^{-2}$$

We can then compare this to the cluster's central surface brightness. The distance modulus of the cluster is

$$(m - M) = 5 \log d - 5 = 14.33$$

After correcting for $A_V = 0.06$ of reddening, the absolute central surface brightness is

$$\mu_V = 16.6 - 0.06 = 16.54 \Longrightarrow \mu_{M_V} = 2.21$$

where μ_{M_V} is in absolute magnitudes per square arcsec. Since the Sun has an absolute V magnitude of 4.83,

$$M = -2.5 \log I_0 + 4.83 \Longrightarrow I_0 = 11.2\, L_\odot \text{ arcsec}^{-2}$$

At a scale of 0.036 pc arcsec^{-2}, this gives $I_0 = 8800 L_\odot$ pc^{-2}. The mass-to-light ratio is therefore $\Sigma_0/I_0 = 1.46$. This is not too difference from that of the Sun (which is 1 by definition), so there is really no evidence for dark matter.

b) (10 pts) Suppose the Sun is orbiting the center of the Galaxy with a circular velocity of 220 km s^{-1}. What does this imply for orbit of M13 about the center of the Milky Way. Is the orbit roughly circular or very elliptical?

The Sun's distance from the Galactic center is about 8 kpc, which is roughly the same as that for M13. If the Sun is orbiting with a circular velocity of 220 km s^{-1}, then to a good approximation, the mass enclosed by the orbit is simply given by

$$\frac{mv^2}{R} = \frac{Gm\mathcal{M}_G}{R^2} \Longrightarrow \mathcal{M}_G = \frac{v^2 R}{G} = 9.5 \times 10^{10} \mathcal{M}_\odot$$

Now, let's consider the tidal radius of M13. The cluster's concentration of $c = 1.53$ and a core radius of 1.3 pc implies a tidal radius of

$$c = \log r_t / r_0 = 1.53 \Longrightarrow r_t = 33.9\, r_0 = 21 \text{ arcmin} = 45 \text{ pc}$$

Again, we can use the distance modulus and reddening to calculate the total luminosity of the cluster

$$M_V = 5.48 - 0.06 - (m - M) = -8.91 \quad \text{and} \quad M_V = -2.5 \log L_T + 4.83$$

which gives $L_T \approx 238,000\, L_\odot$ A mass to light ratio of 1.46 then implies a total cluster mass of $\mathcal{M}_C = 347,000\, \mathcal{M}_\odot$. Since the tidal radius of the cluster should be

$$r_t = R_p \left(\frac{\mathcal{M}_C}{\mathcal{M}_G (3 + \epsilon)} \right)^{1/3} \Longrightarrow \mathcal{M}_G = \left(\frac{R_p}{r_t} \right)^3 \frac{\mathcal{M}_C}{(3 + \epsilon)}$$

where R_p is the orbit's peri-galacticon. If the cluster's orbit were circular, then $R_p = 8400$ pc, $\epsilon = 0$, and the galaxy mass would be $75 \times 10^{10} \mathcal{M}_\odot$. Since this is much greater than the value obtained from the solar motion, it suggests that M13's orbit has significant ellipticity.

8. (10 pts) Is it raining in M87, the central elliptical galaxy of the Virgo Cluster? In other words, is cooler (emission-line) gas condensing out of the cluster's hot x-ray medium and falling into the galaxy? Suppose that near the center of the cluster, the 10^7 K degree x-ray gas has a density of $\sim 10^{-2}$ atoms cm^{-3} (rather than the typical density of $\sim 10^{-3}$ atoms cm^{-3}). Will the gas cool? If the gas remains in pressure equilibrium with its surroundings, what will happen to it?

The cooling time of x-ray gas is given by

$$t_{\text{cool}} = \frac{kT^{1/2}}{2 \times 10^{-27}\, n_e} = 2.2 \times 10^{10} \left(\frac{T}{10^8}\right)^{1/2} \left(\frac{n_e}{10^{-3}\ \text{cm}^{-3}}\right)^{-1} \ \text{yr}$$

For a $T = 10^7$ K gas with $n_e = 10^{-2}$ cm^{-3}, this works out to $t_{\text{cool}} = 7 \times 10^8$ yr. This is not only short compared to a Hubble time, it is also shorter than the timescale to achieve hydrostatic equilibrium

$$t_p = 6.5 \times 10^8 \left(\frac{T}{10^8}\right)^{-1/2} \left(\frac{D}{\text{Mpc}}\right) \ \text{yr}$$

which is $\sim 2 \times 10^9$ yr for a ~ 1 Mpc sized cluster. So the gas cannot be in hydrostatic equilibrium, and will be cooling rapidly. Since the ideal gas law $P \propto \rho T$ applies, if a cooler gas is in pressure equilibrium with its surroundings, it must be denser. Consequently, there will be a buoyancy force on the gas, causing it to fall towards the center of the cluster. It is raining in M87.

9. (10 pts) The "Einstein Cross" is the (quadruple) image of a $z = 1.695$ quasar lensed by the nuclear region of a $z = 0.0394$ galaxy. In the current Planck satellite ΛCDM cosmology, the proper distance to the quasar $d_p = 4706$ Mpc, the proper distance to the lens is 167 Mpc, and the proper distance to the quasar as seen from the lens is 4370 Mpc. Can microlensing by $\sim 1\ \mathcal{M}_\odot$ stars in the bulge of the galaxy resolve the broad line region of the background quasar? (Note: consider for a moment which cosmological distance you need to do this calculation.)

The equation for the Einstein radius is

$$\alpha_0 = \left(\frac{4G\mathcal{M}}{c^2} \cdot \frac{D_{ds}}{D_d D_s} \right)^{1/2}$$

where the distances above represent angular size distances. (We are, after all, dealing with angles on the sky.) So the proper distances of D_d and D_s must be divied by $(1+z)$ (i.e., 1.0394 for the D_d and 2.695 for the quasar), and D_s must be divided by the ratio of the size of the universe at the source to the size of the universe at the lens, i.e.,

$$(1+z)_{ds} = \frac{(1+z)_s}{(1+z)_d}$$

Plugging in the numbers gives an angle of 3.4×10^{-11} radians, or 7.0 microarcseconds. At a proper distance of 4706 Mpc, this angular size corresponds to a linear distance of $r \approx \alpha_0 D_s \approx 0.06$ pc. The size of the broad line region is ~ 0.02 pc. So the stars can not quite resolve the broad line region.